

You are standing on Earth looking due-North. A friend tells you by phone that she is seeing a beautiful aurora, right now, located directly over her head at an altitude, h , of 500 kilometers. You know that the radius, R , of Earth is 6,378 kilometers. In this problem, you will determine whether the aurora will be visible above your horizon as you look North. This same mathematics will also let you answer many other problems too. Can you think of a few?

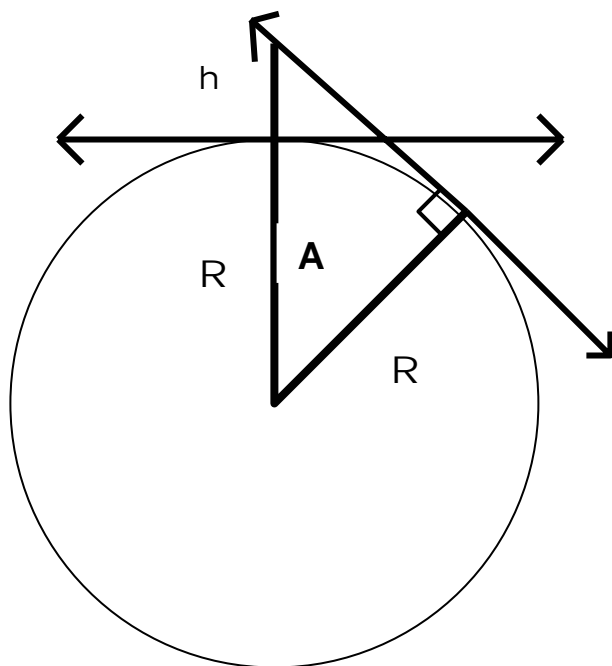
Problem 1: Show that, in the diagram, the line connecting 'You' to the aurora is your horizon line. Draw a line that represents the direction of 'straight up', also called your Zenith.

Problem 2 – The angle, A , represents the difference in latitude between where you are, and where your friend is located, as she views the aurora. Describe, in terms of sines or cosines, the trigonometric relationship between this angle, A , and the height of the aurora, h .

Problem 3 – The aurora was located at an altitude of 500 km, and your friend is located at a latitude of 65 degrees North. At what latitude southward of your friend will the aurora be just at your northern horizon? Will you be able to see it?

Problem 4 – At what latitudes will the aurora be visible above the horizon?

Problem 5 – At the same time as the aurora, your friend sees a bright meteor flash over head at an altitude of 90 kilometers. Could you see this meteor from your latitude calculated in Problem 3?



Suppose there was a plane that was just over your head from one location on Earth. At the same time, another observer located some distance away says that she can see the same plane at the same time, but it is located just above her horizon from where she is standing?

This simple geometric problem lets you determine how far away from some object (an aurora, a meteor, a plane) you will be if you are just seeing it above the horizon.

This problem is solved by using a simple geometric relationship, along with the definition of the 'cosine' of angle A in the right-triangle. The diagram above shows the relevant lengths and angles for an object located ' h ' kilometers above the Earth, and with the radius of Earth, R , defined as 6,378 kilometers.

The first problem has the student determine the formula that relates the height, h , and the sine of the angle, A . The angle, A , is the critical angle A you need to be at to just see some object above the horizon. At smaller angles, the object appears higher up in the sky. Note, the two tangent lines in the figure represent the 'horizons' of two observers viewing the same object, O .

Problem 1: This is your horizon line because it is a line that is exactly tangent to Earth's surface at your location. The line drawn 90 degrees to this line points to your Zenith 'directly over head'.

Problem 2: From the properties of the indicated right triangle: $\cos(A) = R/(R + h)$

Problem 3: $\cos(A) = 6378/(6378 + 500)$ so $\cos(A) = 0.9273$ and so $A = 22$ degrees. Your friend was at 65 degrees North, so your latitude would be $65 - 22 = 43$ degrees North. You will not see the aurora at your latitude because it is exactly AT the horizon.

Problem 4 – At what latitudes will the aurora be visible above the horizon? At latitudes between 43 North and 65 North.

Problem 5 – No, because h is smaller than for the aurora. This means that you would have to be much farther north of where you were standing to see it above the horizon. The critical latitude where you see it AT the horizon would be $65 \text{ degrees} - \arccosine(6378/(6378+90)) = 55.5 \text{ degrees North}$. This is 9.5 degrees North of where you are, which corresponds to a viewing spot $(9.5/360) \cdot 2 \pi (6,378 \text{ km}) = 1,057 \text{ kilometers north of your location}$.